

Smart structural elements for the condition monitoring of bridge structures

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ABSTRACT: This paper introduces a concept of smart structural elements for the real-time condition monitoring of bridges. These are prefabricated reinforced concrete elements embedding a permanent sensing system and capable of self-diagnosis when in operation. The real-time assessment is automatically controlled by a numerical algorithm founded on Bayesian logic: the method assigns a probability to each possible damage scenario, and estimates the statistical distribution of the damage parameters involved (such as location and extent). To verify the effectiveness of the technology, we produced and tested in the laboratory a reduced-scale prototype of smart beam. The specimen is 3.8 m long and has a 0.3 by 0.5 rectangular cross-section, and has been prestressed using a Dywidag bar, in such a way as to control the preload level. The sensor system includes a multiplexed version of SOFO interferometric sensors mounted on a composite bar, along with a number of traditional metal-foil strain gauges. The method allowed to clearly recognize increasing disease states, simulated on the beam by gradually reducing the prestressing level.

1 INTRODUCTION

Fiber optics technology today offers durable solutions for bridge monitoring, and recent advances in Micro Optic Electro Mechanical Systems (MOEMS) suggest that in the near future we will be able to rely on very small-scale optical devices. These innovations will radically change monitoring methods for civil structures in coming years, although the authors suggest that industrial deployment is essential to overcome the cost issue. In this light, the University of Trento is promoting research to develop construction systems for smart elements suitable for wide application (Zonta *et. al* 2006, 2007). These elements are pre-cast Reinforced Concrete (RC) members embedding a sensing system and capable of self-diagnosis. The technology particularly targets new bridges and includes strain and environmental sensors.

A major issue is how to exploit appropriately the large amount of measurements recorded by the system. A Bayesian approach provides a rational framework to interpret measurement data while also allowing proper handling of all prior knowledge, including material properties, environmental conditions and sensor performance. This methodology allows to identify not only the most likely values of the unknown damage parameters (such as type, position and extent) but also their posterior probability distribution.

This paper introduces both the technological and the methodological aspects of the development and operation of the smart elements, and is essentially divided into two parts: first, in Section 2, we present the general formulation of the Bayesian algorithm developed to assess the condition state of the smart elements; next, in Section 3, we illustrate the development of a reduced-scale prototype of smart beam and we demonstrate how the updating algorithm applies to the identification of the loss of prestressing, artificially produced on the element during a laboratory experiment.

2 IDENTIFICATION CONCEPT

2.1 Introduction to Bayesian logic

The Bayesian theory of probability originates from Bayes' well known essay (Bayes 1763). Many modern specialised textbooks can provide the reader with a critical review and applications of this theory to data analysis (see for instance Gregory 2005, Sivia 2006). Of all the papers dealing with application of Bayesian theory to engineering problems, the authors wish to underline Beck's work (Papadimitriou *et al.* 1997, Beck & Katafygiotis 1998, Beck & Au 2002), which by disseminating these concepts had great impact on the civil engineering community.

2.2 Problem statement

Assume we have a bridge instrumented with a certain number of sensors, and we want to gain information on the state of the bridge based on the data recorded. Each sensor provides a measurement for each of N_T time values $(t_1, t_2, \dots, t_{N_T})$. It is convenient to distinguish two types of gauges: sensors recording the structural response of the bridge, and sensors recording the load and environmental effects on the bridge. The first set might include sensors such as strain gauges, accelerometers, displacement transducers. In a broad sense, we can classify as a *response sensor* any instrumentation setup capable of providing a response quantity. For simplicity's sake, here we will assume that these are all strain gauges. Say that the structure is instrumented with N_s sensors of this type, labelled $(s_1, s_2, \dots, s_{N_s})$, and let us label $\varepsilon_{i,j}$ the strain measurement recorded by sensor s_j at time t_i . The second set includes, for example, thermocouples installed next to the strain gauges for temperature compensation or load cells applied at the bridge bearing, capable of recording the traffic loads: these are defined as *environmental sensors*. The basic idea is that response measurements depend, on the one hand, on external actions such as temperature and loads; on the other on long term effects, such as dead load redistribution, creep, shrinkage, strand relaxation. Long term effects produce slow changes in the structural response. Accordingly, it is convenient to organize measurements into time intervals (e.g. per day), it being a time span short enough to assume that long term changes are negligible and long enough to ensure that short term changes are significant. Let us define $\mathbf{m}_{T,j}$ the vector including all the strain measurements recorded by the sensor s_j in time interval T and \mathbf{m}_T the matrix including all strains in time interval T . Finally let us label \mathbf{M}_T the whole dataset from the first time interval (i.e. from the start of monitoring) to time interval T . Similarly to strain measurements, we define as \mathbf{h}_T the matrix including all environmental measurements in time interval T . Data acquired during this sample period can be organized into matrix form:

$$\mathbf{m}_{T,j} = \varepsilon_{T,j}^0 + \mathbf{h}_T \mathbf{a}_{T,j} + \mathbf{g}_{T,j} \quad (1)$$

where $\varepsilon_{T,j}^0$ is strain independent of load or temperature (i.e. the compensated strain), $\mathbf{a}_{T,j}$ is the vector including the coefficients of the linear correlation from load to strain, and $\mathbf{g}_{T,j}$ is a noise vector, which is assumed to have zero mean Gaussian distribution with standard deviation $(\sigma_g)_{T,j}$. Equation 1 assumes that the effect of load (or in general of external actions) on strain is linear, although this is not the most general case.

We assume that the presence of damage in the structure will generally modify the compensated response history. Thus, the identification method seeks to detect the presence of damage by comparison of the compensated measurements with the theoretical response produced by a model. In practice, it is convenient to divide the domain of the possible structural response into a mutually exclusive and exhaustive set of scenarios $(S_1, S_2, \dots, S_{N_d})$, each defining the structural behaviour in a specific condition (e.g. reinforcement corrosion). A single probability result for each scenario is one of the main advantages of this approach. The structural strain response ${}^n r_{T,j}(\mathbf{x})$ for day T and sensor s_j in scenario n is controlled by a certain number of parameters ${}^n \mathbf{x}$ (e.g. damage position, activation time, corrosion rate). The structural response is completely defined by specifying a scenario and a value for the correlated parameter set. Here, as in the Bayesian model selection theory (Bretthorst 1996, Sivia 2006), the discrete meta-parameter scenario identifies the response function which in turn is specified by a parameter set. The difference between measurements and structural response is just random noise. Assuming scenario n and ${}^n \mathbf{x}$ to be correct, the compensated strain can be expressed as:

$$\varepsilon_{T,j}^0 = {}^n r_{T,j}({}^n \mathbf{x}) + e_{T,j} \quad (2)$$

where $e_{T,j}$ is the model error. We assume $e_{T,j}$ is a random error modelled as an uncorrelated Gaussian noise, with zero mean and standard deviation $(\sigma_e)_{T,j}$. This is independent of time, but generally changes with the sensor. Evidently, $(\sigma_e)_{T,j}$ changes with sensor type but we may expect a dependency, for example, on sensor position or precision. More in general, we can state that $(\sigma_e)_{T,j}$ is a function of a set of scenario-dependent parameters \mathbf{y} . In summary, we can state that each scenario is fully described by a set of parameters $\mathbf{p}=[\mathbf{x} \ \mathbf{y}]$.

2.3 Compensation of strain measurements

Estimation of the compensated strain $\varepsilon_{T,j}^0$ introduced in Equation 1 is defined by its mean value $(\mu_\varepsilon)_{T,j}$ and standard deviation $(\sigma_\varepsilon)_{T,j}$. As Equation 1 is linear, it is possible to formalize a rigorous Bayesian procedure to identify these quantities. To do so, one must furnish a prior characterization of the linear correlation vector $\mathbf{a}_{T,j}$ by distribution PDF($\mathbf{a}_{T,j}|I$)=Normal($\mathbf{a}_{T,j};\boldsymbol{\mu}_{\mathbf{a}},\boldsymbol{\Sigma}_{\mathbf{a}}$), where Normal($\mathbf{x};\boldsymbol{\mu},\boldsymbol{\Sigma}$) indicates the multi-dimensional Gaussian distribution with mean value vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$ computed in \mathbf{x} , I means all the available background information and PDF stands for Probability Density Function. Then the procedure proceeds as follows:

- Expanded vector $\mathbf{v}_{T,j}=[\mathbf{a}_{T,j}^T \ \varepsilon_{T,j}^0]^T$ is defined, and mean value $\boldsymbol{\mu}_{\mathbf{v}}$ and covariance matrix $\boldsymbol{\Sigma}_{\mathbf{v}}$ of its prior distribution are obtained by adding zero components to $\boldsymbol{\mu}_{\mathbf{a}}$ and $\boldsymbol{\Sigma}_{\mathbf{a}}$.
- The best fitting value for $\mathbf{v}_{T,j}$ is calculated by solving a linear system and assumed as mean value $\boldsymbol{\mu}_{L_v}$ of the likelihood distribution, $(\sigma_g)_{T,j}$ is derived by the difference between the actual measurements and the best fitting prediction (Yuen 2002), and the covariance matrix $\boldsymbol{\Sigma}_{L_v}$ is given by the classical least squares formula (Gregory 2005).
- Since posterior distribution is the product of prior and likelihood, both of which are Gaussian, mean value $\boldsymbol{\mu}_{P_v}$ and its covariance matrix $\boldsymbol{\Sigma}_{P_v}$ can be computed in close form (Jaynes 2003).
- Once the posterior distribution of expanded vector $\mathbf{v}_{T,j}$ is obtained, $(\mu_\varepsilon)_{T,j}$ and $(\sigma_\varepsilon)_{T,j}$ derive from its marginalization and therefore, since distribution is Gaussian, they can be directly read as components of $\boldsymbol{\mu}_{P_v}$ and $\boldsymbol{\Sigma}_{P_v}$ respectively.

The whole procedure can be seen as a kind of *filter* that processes the response ($\mathbf{m}_{T,j}$) and the environmental ($\mathbf{h}_{T,j}$) measurements to give an estimation of the compensated strain, i.e. the strain independent from environment influences. By doing so, the number of samples is reduced, so that a single strain estimation is associated to each sensor and time interval. Pros of the filter are twofold: on one hand it reduces the number of data that have to be processed by the general identification model described in the next section while on the other it allows to avoid including in that model the any environmental effects that the filter annihilates.

As anticipated, to apply the filter it is necessary to fix a criterion to select distribution PDF($\mathbf{a}_{T,j}|I$). It is worth noting that, at the end of the procedure previously described, it is easy to read into the components of $\boldsymbol{\mu}_{P_v}$ and $\boldsymbol{\Sigma}_{P_v}$ the ones related to $\mathbf{a}_{T,j}$. Thus, the updated distribution for $\mathbf{a}_{T,j}$ can be assumed as a prior for $\mathbf{a}_{T+1,j}$: this is logical if one assumes that $\mathbf{a}_{T,j} = \mathbf{a}_{T+1,j}$. Alternatively, when the value of \mathbf{a} evolves in time, we assume that:

$$\mathbf{a}_{T+1,j} = \mathbf{a}_{T,j} + \mathbf{d}\mathbf{a}_{T,j} \quad (3)$$

where $\mathbf{d}\mathbf{a}_{T,j}$ models the variation of linear correlation vector from T to $T+1$. If prior distributions for $\mathbf{a}_{T,j}$ and for $\mathbf{d}\mathbf{a}_{T,j}$ are both Gaussian, it is straightforward to calculate the prior distribution for $\mathbf{a}_{T,j+1}$ as that related to the sum of Gaussian distributed random variables.

2.4 Scenario updating procedure

Once compensated strain is estimated, the method here presented allows for calculation of the probability of being in each scenario, as well as the statistical distribution of the associated parameters. If the probability at day (or time interval) $T-1$ of being in a specific scenario is known, Bayes' theorem allows us to update this probability using the fresh data acquired on day T :

$$\text{prob}(S_n | \mathbf{M}_T, I) = \frac{\text{PDF}(\mathbf{m}_T | \mathbf{M}_{T-1}, S_n, I) \cdot \text{prob}(S_n | \mathbf{M}_{T-1}, I)}{\text{PDF}(\mathbf{M}_T | \mathbf{M}_{T-1}, I)} \quad (4)$$

The first term at the numerator, sometimes referred to as *evidence* of scenario S_n , can be calculated by integrating over the whole parameter domain $D^n \mathbf{p}$, using:

$$\text{PDF}(\mathbf{m}_T | \mathbf{M}_{T-1}, S_n, I) = \int_{D^n \mathbf{p}} \text{PDF}(\mathbf{m}_T | \mathbf{p}, S_n, I) \cdot \text{PDF}({}^n \mathbf{p} | \mathbf{M}_{T-1}, S_n, I) \cdot d^n \mathbf{p} \quad (5)$$

When many parameters are involved in Equation 5, the exact integration might require an exceptional computational effort and needs to be circumvented with numerical techniques. One of the simplest ways is to apply Laplace asymptotic expansion (Beck & Katafygiotis 1998). Monte Carlo algorithms are alternative methods: they are classified into Classical Monte Carlo methods and Markov Chain Monte Carlo methods (MacKay 2003); both rely on the possibility of drawing samples from a target distribution and of computing an integral making average along the sample. While the former family draws samples independently from a fixed distribution, the latter produces a step by step random walk in the parameter domain. Whatever the numerical method adopted, the denominator of Equation 4 can be expressed as:

$$\text{PDF}(\mathbf{M}_T | \mathbf{M}_{T-1}, I) = \sum_{n=1}^{Nd} \text{PDF}(\mathbf{m}_T | \mathbf{M}_{T-1}, S_n, I) \cdot \text{prob}(S_n | \mathbf{M}_{T-1}, I). \quad (6)$$

In a similar manner, we can calculate the PDF of the parameters governing a specific scenario:

$$\text{PDF}({}^n \mathbf{p} | \mathbf{M}_T, S_n, I) = \frac{\text{PDF}(\mathbf{m}_T | \mathbf{p}, S_n, I) \cdot \text{PDF}({}^n \mathbf{p} | \mathbf{M}_{T-1}, S_n, I)}{\text{PDF}(\mathbf{m}_T | \mathbf{M}_{T-1}, S_n, I)}, \quad (7)$$

where the first term at the numerator, which appears also in Equation 5, is:

$$\text{PDF}(\mathbf{m}_T | \mathbf{p}, S_n, I) = \prod_{j=1}^{Ns} \text{PDF}(\mathbf{m}_{T,j} | \mathbf{p}, S_n, I). \quad (8)$$

Of course, the first application of Equations 4 and 7 requires the definition of prior values of probability of each scenario $\text{prob}(S_n | I)$ and distribution of parameters vector $\text{PDF}({}^n \mathbf{p} | S_n, I)$. The only remaining problem is how to obtain $\text{PDF}(\mathbf{m}_{T,j} | \mathbf{p}, S_n, I)$. It is demonstrated that:

$$\text{PDF}(\mathbf{m}_{T,j} | \mathbf{p}, S_n, I) = \text{Normal}({}^n r_{T,j}(\mathbf{x}); (\mu_\epsilon)_{T,j}, (\sigma_\epsilon)_{T,j}^2 + ({}^n \sigma_\epsilon)_j^2), \quad (9)$$

In summary, the proposed Bayesian identification procedure consists of two steps: first, the strain measurements are compensated using the filter illustrated in Section 2.3; then the identification procedure presented in this section processes the obtained estimation.

3 LABORATORY VALIDATION

3.1 Specimen description and test protocol

To illustrate how the Bayesian updating procedure works, here below is the application of the algorithm in a laboratory campaign, in which a reduced-scale sample of a precast RC element has been produced and tested to simulate the response of the deck of new bridges. The specimen is 3.8 m long, has a 0.3 by 0.5 rectangular cross-section and has been pre-stressed using an instrumented Dywidag bar (Fig. 1), in such a way as to control the preload level.

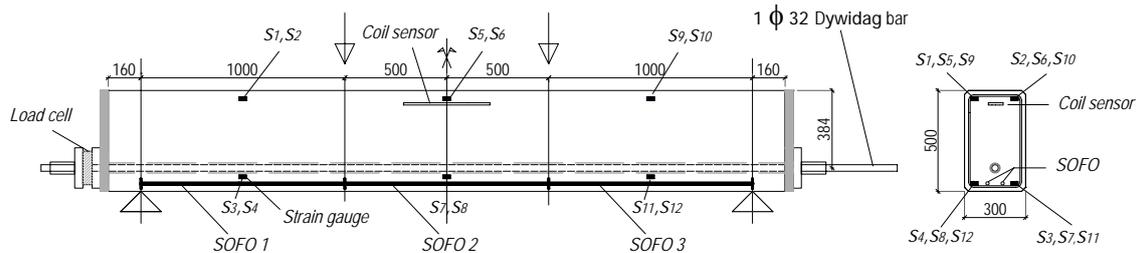


Figure 1. Longitudinal and cross sections of the prototype of smart element.

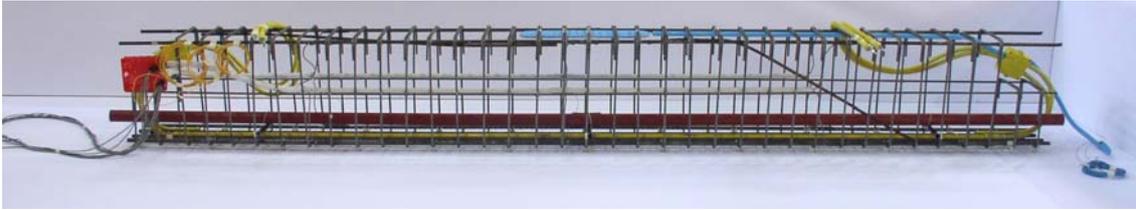


Figure 2. Instrumented reinforcement frame before concrete casting.

The prototype is equipped with traditional sensors, including 12 metal-foil strain gauges and 2 thermocouples, as well as with two novel types of Fiber Optic Sensors (FOS). As shown in Figure 1, the strain gauges measure the longitudinal strain and are arranged at the four corners of 3 cross sections. The first type of FOS is a multiplexed version of the standard SOFO (Surveillance d'Ouvrages par Fibres Optiques) interferometric sensor, originally developed, produced and commercialized by Smartec SA (Pozzi *et al.* 2007). It is arranged in a 3-field scheme, placed along the lower edge of the specimen with a measurement base of 1.00 m for each field. To facilitate the sensor installation into the reinforcement frame and to protect it during concrete pouring, its packaging makes use of a composite support to which the fibers are glued. The second type of FOS, labelled Coil sensor, is based on the direct time-of-flight measurement of pulses travelling into the fiber. It is placed at the upper edge of the mid span section and measures the strain on a 0.60 m base. Figure 2 shows the instrumented reinforcement frame before concrete pouring: the coloured boxes at the ends of the frame were placed at the interface between concrete and mould and used to introduce the transmission cables (Fig. 3).

The scope of the experiment was to correlate the response of the embedded sensor to different prestressing levels induced on the beam. Different levels of cracking have been produced through the application of a vertical load action using an hydraulic actuator (Fig. 3): the load protocol includes a sequence of load-unload cycles, repeated for different values of prestressing, up to the yielding of the regular reinforcement. In this case, the specimen was subjected to 6 cycles of the vertical load, each ranging from 0 up to 250 kN. Between successive cycles the prestressing force was progressively decreased from 450 kN down to 250 kN. During this test, each sensor was continuously acquired with a sample frequency of 1 Hz. Figure 4 schematically shows the load protocol as well as that of the grouping of the measurements into time intervals.

3.2 Identification procedure

Aim of the identification is to recognize the loss of prestressing based only on the strain measurements and vertical load data, and to compare the resulting estimation with that measured by the load cell placed on the bar. The 12 foil strain gauges are assumed to be response sensors (s_{1-12}), while the load cell that records the vertical load is the only environmental sensor adopted.

As illustrated in Section 2, data elaboration is carried out in two steps: first the complete strain history is deperated of the environmental contributions (i.e. of the influence of the vertical load) to obtain a compensated strain history, then this is processed to obtain the prestressing loss.

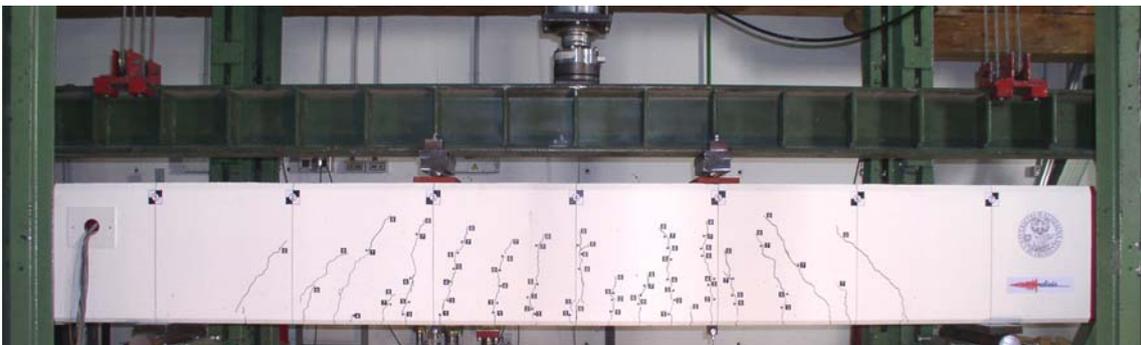


Figure 3. Appearance of the prototype during the loading test.

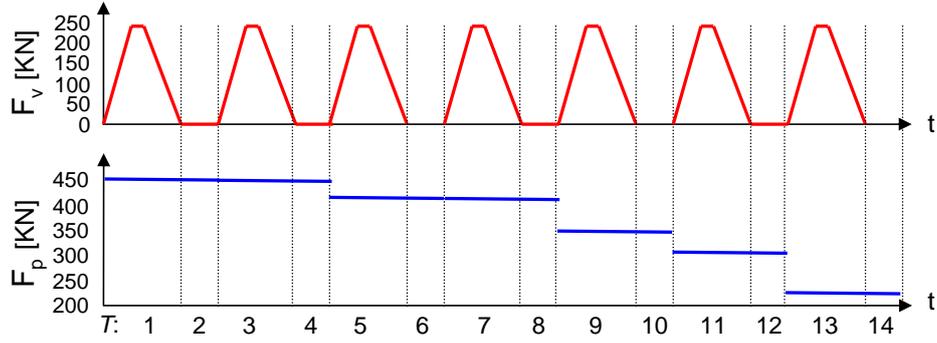


Figure 4. Load protocol: vertical load (upper graph) and prestressing load (lower graph).

To apply the first step, the whole time history is divided into 14 time intervals (Fig. 4), so that each interval includes either a complete loading sequence or a period with no vertical load applied. The filter described in Section 2.3 is applied, independently to each response sensor, obtaining an estimation of the compensated strain value for each time interval. To do so, the prior distribution for the linear correlation coefficient $a_{T+1,j}$ is assumed to have the same mean value of that updated for $a_{T,j}$ but with double the variance: according to the formalism of Equation 3, this means that $\sigma^2(\mathbf{d}a_{T,j}) = \sigma^2(\mathbf{a}_{T,j})$. This assumption takes into account the possibility of a stiffness variation due to the cracking of concrete. As an example of results, Figure 5a reports both the strain measurements (with dots) and the compensated strain history for sensor s_8 . To depict the compensated history, 3 lines are shown in the picture: the continuous one indicates the mean values $(\mu_\varepsilon)_{T,8}$ while the dash-dotted and dotted ones add and subtract the standard deviation $(\sigma_\varepsilon)_{T,8}$ to the mean value.

Once the compensated strain history is obtained, two possible scenarios are assumed: according to the first one (S_1), no loss of prestressing is involved while, according to the second one (S_2), a loss of prestressing of arbitrary amount is assumed. The compensated strain is transformed into the difference $\Delta\varepsilon^0_{T,j}$ between the current state and the reference one, i.e. the state before any vertical load was applied. In detail, S_1 models this difference as due only to plastic effects:

$$\Delta\varepsilon^0_{T,j} = \varepsilon p_{T,j} + e_{T,j} \quad (10)$$

where $\varepsilon p_{T,j}$ is the plastic contribution due to creep, relative to sensor s_j and time interval T , and is modelled as a deterministic function (Collins & Mitchell 1991). Thus, no free parameter \mathbf{x} is assumed for S_1 , and ${}^1r_{T,j} = \varepsilon p_{T,j}$. For this and for the following scenario, prior knowledge on error variable $e_{T,j}$ is assumed to be a zero mean Gaussian distribution, whose variance is unknown and modelled by a flat distribution between 10 and 40 $\mu\varepsilon$. Scenario S_2 assumes a linear model for the prestressing contribution, maintaining the same formula for the plastic effect. The difference of compensated strain here is modelled as:

$$\Delta\varepsilon^0_{T,j} = k_j \cdot \Delta P_T / E_c + \varepsilon p_{T,j} + e_{T,j} \quad (11)$$

where k_j is a geometrical constant (known with certainty) relative to sensor s_j , E_c is the Young modulus of concrete, ΔP_T is the loss of prestressing at time interval T . Based on an independent test on concrete specimens, E_c is assumed to be modelled by a Gaussian distribution with mean value 20 GPa and standard deviation 5 Gpa. Conversely, a non-informative flat distribution is assumed for ΔP_T . Assuming the formalism of Section 2.4, the parameter vector and the scenario response function for S_2 are respectively:

$${}^2\mathbf{x} = [E_c \Delta P_1 \dots \Delta P_{14}], \quad {}^2r_{T,j}({}^2\mathbf{x}) = k_j \cdot \Delta P_T / E_c + \varepsilon p_{T,j} \quad (12)$$

Figure 5b reports the updated distribution of ΔP_T after processing the data. Continuous line indicates the mean value, while dash-dotted and dotted lines add and subtract the standard deviation. In the same graph the prestressing loss measured by the load cell is reported with circles. Prior probabilities of the two scenarios are assumed to be the same, i.e. $\text{prob}(S_1) = \text{prob}(S_2) = 0.5$. Figure 5c reports the probability of scenario S_2 as more and more estimations become available: as shown, probability grows quickly up to 1, and the identification system becomes sure that a loss of prestressing is involved. The graph shows that during the

first cycles, when the prestressing force was not reduced yet and its value was fluctuating because of external influences, the identification ranges between 0.3 and 0.9. However, once the prestressing force is actually decreased, the system recognizes this variation.

It is worth noting that the two scenarios present a different degree of complexity: scenario S_2 involves free parameters, which make it capable of following more closely the compensated strain history. Furthermore, scenario S_1 can be regarded as a special sub-case of scenario S_2 , when all ΔP_T are null. The reader might argue that, because of this, the probability of scenario S_2 will be always greater than that of S_1 . Actually, this is not necessarily the case. Indeed, by tuning the parameters of scenario S_2 , we can obtain a better fitting than that related to scenario S_1 . However, according to Bayesian logic, also the ratio between the best fitting and the average fitting (the so-called *Ockham factor*) plays a fundamental role (Sivia 2006).

Finally, the limits of the presented application should also be mentioned. According to its basic assumptions, analysis of parameters $a_{T,j}$ is not included in the estimation of the loss of prestressing. In reality, the value of $a_{T,j}$ generally depends on stiffness variation and, consequently, on the cracking pattern induced by the loss of prestressing. During the test, a decrement of the value of $a_{T,j}$ was actually observed. However, far more complex non-linear models are required to describe these interactions.

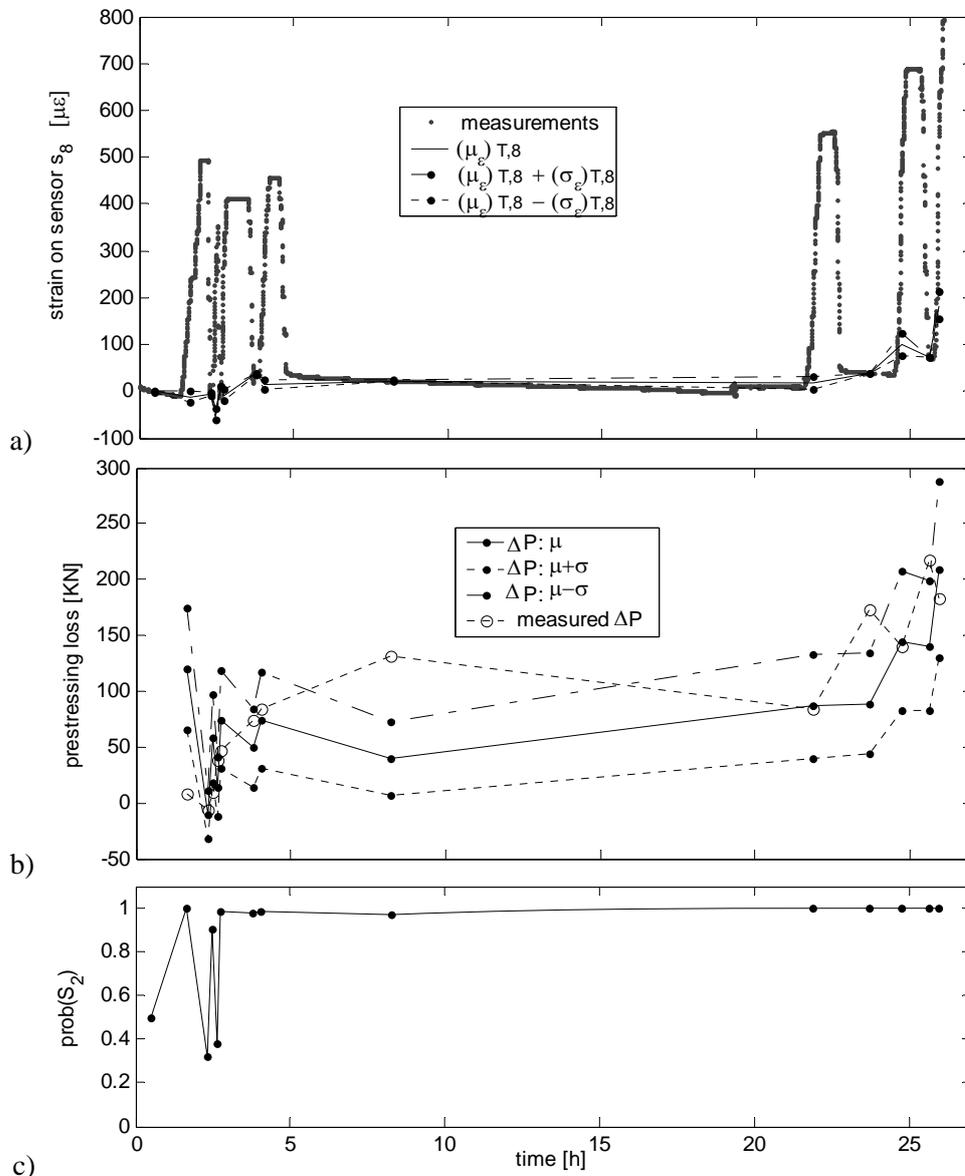


Figure 5. Measured and compensated strain for sensor s_8 (a); estimated and measured prestressing loss (b); probability of scenario S_2 (c).

4 SUMMARY

The proposed Bayesian identification procedure provides a rational quantification of the influence of monitoring data on the knowledge of the occurrence of different scenarios. With respect to classical damage detection methods, its merit is to provide not only information on the damage, but also the degree of confidence of this information. This is of paramount importance when the results of damage assessment serve as input in decision-making processes.

The application of this procedure to the condition assessment of a smart element prototype shows the potential of this approach. For instance, the test reported clearly shows that parameters such as loss of prestressing can be identified with a high degree of reliability. Despite the fact that the example provided is limited, the general approach is not problem dependent, and can be extended to a broader class of problems, including manifold scenarios, model or material uncertainties, prior knowledge of parameter distribution.

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